OPTIMAL STRUCTURE FOR A CO-OPERATIVE NUCLEUS BREEDING SYSTEM

N. JACKSON* and HELEN NEWTON TURNER*

Summary

Genetic gains in an established co-operative nucleus breeding system are maximized when the number of nucleus ewes is between 5 and 10 per cent of the total number of flock ewes, and nucleus ewe replacements are approximately half nucleus- and half flock-born. The optimum age structure (at a 70 per cent lambing rate) is 2 and 3 ram age-groups in the nucleus and flocks respectively, and 4 and 5 ewe age-groups, respectively. The total number of ewes in the system, and the number of flocks over which they are spread, do not influence genetic gains provided flocks or nucleus size is not small.

I. INTRODUCTION

In the Australian Merino sheep industry, sires for commercial flocks have in the past been bred in studs (nuclei), many of which have been closed (Short and Carter 1955). Genetic gains in a closed nucleus, and hence in its dependent flocks, have been discussed by various authors, and reviewed by Turner and Young (1969).

This paper will discuss genetic gains obtainable in flocks dependent on a nucleus which is not closed; in particular, the transfer of superior ewes from flocks to nucleus will be considered. This scheme is “co-operative” because one or more flockowners may be involved. Morley (1952) studied a non-co-operative nucleus system within a single flock. Many co-operative nucleus systems are now in operation (Dun and Eastoe 1970; Hight and Rae 1970; Jefferies 1970; Rae and Hight 1969; Roberts personal communication; and Shepherd 1971), and general guidelines have been given, but there have been no theoretical studies of the comparative efficiency of various structures. Only some aspects can be considered here, and other studies will follow.

II. METHODS

The structure examined is specified by the variables in Table 1, Figure 1 giving an illustrative example. A range of values was given to each variable and to the vital statistics of Table 2; the combination giving greatest efficiency was sought.

Efficiency was defined as the expected annual genetic gains in the co-operating flocks in a stable system. As stability was defined as being reached when the
Fig. 1.—A nucleus breeding system of optimal structure for 20,000 flock ewes.
E = 1000
NUCLEUS EWE BREEDERS
(a = 4 AGE GROUPS)

E/4 = 250 EWES C.F.A.

\[ \text{dE} = \text{25 REPLACEMENTS BORN IN NUCLEUS} \]
\[ \text{dE} = \text{225 REPLACEMENTS BORN IN FLOCKS} \]

\[ \text{dE} = \text{108 CULLED} \]
\[ \text{dE} = \text{225 CULLED} \]
\[ \text{dE} = \text{NO CULLED} \]

\[ a \text{ AR FN} = \text{125} \text{ CULLED} \]
\[ a = \text{TOP 36%} \]

\[ \text{dF} = \text{3775 FLOCK REPLACEMENTS} \]
\[ \text{dF} = \text{NEXT 64%} \]

\[ (1-a-b) \text{ FN} = \text{3500 CULLED} \]

\[ d \text{ FN} = \text{225 REPLACEMENTS BORN IN NUCLEUS} \]

\[ \text{dF} = \text{3775 REPLACEMENTS BORN IN FLOCKS} \]

\[ \text{FN} = \text{40000 C.F.A. EWES} \]

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TABLE 1
Variables which describe the structure of a nucleus breeding system

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbols used for this variable in Nucleus</th>
<th>Conditions to be satisfied for viability of the system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of flocks</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Number of breeding ewes</td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>Ratio of rams to ewes</td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>Number of age groups; rams ewes</td>
<td>r</td>
<td>s</td>
</tr>
</tbody>
</table>

Proportion of each flock ewe drop which:
- Transfer to nucleus: $a$ \( a + b \geq 1 \)
- Remain in flock: $b$
- Are culled: \((1-a-b)\)

Proportion of nucleus ewe drop which:
- Remain in nucleus: \(c\)
- Transfer to flocks: \(d\)
- Are culled: \((1-c-d)\)

Proportion of nucleus ram drop which:
- Remain in nucleus: \(e\)
- Transfer to flocks: \(f\)
- Are culled: \((1-p-q)\)

Genetic difference between nucleus and flocks before selection in year 1: \(I_p\)

* These variables may all be calculated from others, when definitive values have been given.

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TABLE 2
Vital statistics for nucleus and flock sheep

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Symbol used</th>
<th>Range of values examined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of lambs of each sex born and reaching joining age, per ewe joined in (i)th age group of ewes ((i = 1) for 0-11 months (= 2) for 12-23 months etc.) (\text{total lambing rate} = 21_i)</td>
<td>(I_i)</td>
<td>low lambing rate* (0.00, 0.25, 0.32, 0.40, 0.43, 0.45, 0.42, 0.38, 0.35, \text{for } i = 1 \text{ to } 9) and high lambing rate (0.00, 0.40, 0.50, 0.62, 0.65, 0.67, 0.60, 0.57, 0.55) (\text{see Turner 1963})</td>
</tr>
<tr>
<td>Death rate for ewes of (i)th age group</td>
<td>(u_i) (0.00 \text{ all } i) and (0.10 \text{ all } i)</td>
<td></td>
</tr>
<tr>
<td>Death rate for rams of (i)th age group</td>
<td>(t_i) (0.00 \text{ all } i) and (0.10 \text{ all } i)</td>
<td></td>
</tr>
<tr>
<td>Age of rams and ewes at first lambing</td>
<td>(k) (k) (2)</td>
<td></td>
</tr>
</tbody>
</table>

* The terms low and high "lambing rate" are used in the text when referring to the two sets of figures presented in this table.
genetic difference between nucleus and flocks was constant from year to year (see Appendix 1), annual genetic gains in nucleus and flocks were then the same, and simply “genetic gains” will be referred to. The optimal structure for an established system was being sought; the method of establishment will be investigated in a later paper.

Production was defined as one normally distributed trait; it might be a single trait, e.g. wool weight, or an index combining several.

Truncation selection for production was practised, at one age (prior to first joining), among nucleus-born rams and ewes and flock-born ewes.

Annual genetic gains in nucleus and flocks were predicted (Appendix 1) for the years necessary to reach stability. Most structures were stable in 4 to 10 years, depending on the initial genetic difference between nucleus and flocks. (Genetic differences among flocks were assumed to be zero.)

Results are expressed in units of:

<table>
<thead>
<tr>
<th>Expected annual genetic gain in standard units</th>
<th>Heritability</th>
</tr>
</thead>
</table>

III. RESULTS AND DISCUSSION

Figure 1 takes as an example a system involving:
- Total of 20,000 breeding ewes in co-operating flocks,
- Lambing rate 0.70 (low lambing rate of Table 2)
- Death rate in adults 0 per cent
- Ram:ewe ratio 1: 50 (2 per cent working rams)

For this system, genetic gains were greatest when:

(i) The number of nucleus ewes was between 5 and 10 per cent of the total number of flock ewes.
(ii) The nucleus ewe replacements were approximately 50 per cent nucleus- and 50 per cent flock-born.
(iii) The numbers of age-groups of rams in the nucleus (r) and flocks (s) were 2 and 3 respectively, and of ewes (e and f), 4 and 5 respectively.
(iv) All nucleus-born ewes not required as nucleus replacements were transferred to the flocks (c + d = 1).

The influence of some variables in Table 1 and Table 2 on genetic gains will be discussed in turn.

(a) Composition of FN

It can readily be seen that the total FN is the deciding factor, not any varying values of F and N, provided that the proportions of ingoing and outgoing sheep are the same in all flocks, and that the flocks are of equal genetic value. This will hold even if flocks are unequal in size. Figure 1, for simplicity, has been drawn with one flock of 20,000 ewes; it could equally have been 10 x 2000, or 20 x 1000, and so on.

(b) Nucleus size relative to total number of flock ewes (E/FN)

The value of this ratio giving greatest genetic gains was between 5 and 10 per cent at both the low and high lambing rates. 5 per cent has been taken for illustration; differences in gains between \( \frac{E}{FN} = 5 \) per cent and 10 per cent are small, and the lower figure minimizes ram wastage.
(c) Proportion “a” of each flock ewe drop transferred to nucleus

The optimal value of “a” was such that approximately 50 per cent of nucleus ewe replacements were flock-born. This percentage will be referred to as the “degree of non-closure” of the nucleus.

Figure 2 compares various levels of $E/FN$ at three “a” values, corresponding to 0, 50 and 90 per cent of non-closure. (Death rates 0, numbers of ram and ewe age groups as in Figure 1).

In the Australian Merino industry as described by Short and Carter (1955), only 2 per cent of all ewes were in registered studs, and there was little or no provision for transfer from lower to higher ranks of the hierarchy (zero non-closure). The industry was therefore operating at sub-optimal values both of $E/FN$ and degree of non-closure.

The structure illustrated in Figure 1 ($E/FN = 5$ per cent, 50 per cent non-closure) had expected annual genetic gains of 0.51 units. The industry structure described by Short and Carter, which has obtained until recently, would have an expected annual genetic gain in production of 0.46 units (Figure 2), that is, would be operating at 90 per cent of maximal genetic gain — if selection were based directly on production.

(d) Optimal age structures

These were dependent on lambing rate, smaller numbers of age-groups giving greater gains at higher rates (Figure 3, see also Turner and Young, Chap. 16.) Among age structures, that of nucleus ewes influenced gains most; age-structures in the flock were less important, 4–5 groups of ewes or 1–3 of rams giving greatest gains but decreases with greater numbers being slight.

(e) Size of the breeding system

This may influence genetic gains through (1) the effect of finite population size in lowering the selection differential, (2) inbreeding.

Finite population size only becomes important when the number of animals of each sex available for selection falls below 50, in the nucleus or any co-operating flock. With a lambing rate of 70, this means $E$ or $F < 150$.

Degree of inbreeding depends on the size of the system and, in the nucleus, on the degree of non-closure. For the system in Figure 1, its effect would be negligible.

System size is unlikely to be important in Australia for the Merino, but could be for other breeds.

IV. PRACTICAL CONSIDERATIONS

The theoretical genetic gains discussed in this paper give a basis for further analyses, which will involve balancing costs and gains. The structure analysed here raises two practical points:

(i) All young ewes in the flocks must be measured, for direct entry into the nucleus; costs of various levels of measurement must be balanced against gains.
Fig. 2.—The effect of relative nucleus size and percentage non-closure on annual genetic gains in a stable nucleus breeding system. Each graph gives expected genetic gains for the breeding system illustrated in Figure 1, for various relative nucleus sizes and various percentages of non-closure, other variables being at their optimal values.

- 0, 0 per cent non-closure; ●, 50 per cent non-closure; Δ, 90 per cent non-closure.
Fig. 3.—The effect of age structure on annual genetic gains in the nucleus when the system is stable. Each graph gives expected gains for the system of Figure 1, for various nucleus age structures, other variables being at their optimal values.

0—0 low lambing rate (70); •••••• high lambing rate (110).
(ii) The optimal ratio of $E/FN$ for genetic gains will provide a ram number in excess of flock requirements. If these cannot be profitably sold, then a ratio which is sub-optimal for genetic gains might be optimal for economic return.

These points will be considered in later papers; this paper gives the genetic gains which will be required in such analyses.

V. CONCLUSIONS

Genetic gains in a partially-closed nucleus system are greater than those in a closed one. The greatest annual genetic gains are obtained if the number of ewes in the nucleus is approximately 5 per cent of the number in the flocks and if about 50 per cent of ewe replacements in the nucleus are drawn from the nucleus, 50 per cent from the flocks.

A co-operative breeding system would be useful for any sheep breed, and would help overcome the problems of small stud size encountered with some British breeds.

APPENDIX 1

The expected annual genetic progress in year $g$ through selection, in the nucleus or flocks, in phenotypic standard deviations is $ih^2/L$, where $i$, $h^2$, and $L$ are selection differential, heritability and average generation interval respectively.

The generation intervals are $(k + 1/4 r (or s) + 1/4 e (or f) - 1/2)$ for nucleus and flock respectively. The selection differentials are composites of the separate selection differentials corresponding to the selected proportions $p, q, a, b, c,$ and $d$ and are:

$$i_n(p) = \frac{i}{4} h a + \frac{i}{4} c \left( \frac{1}{1 + p} \right) + \left( i_a - \frac{p}{1 + p} \right) \left( \frac{L}{1 + p} \right)$$

$$i_f(q) = \frac{i}{4} h b + \frac{i}{4} c \left( \frac{1}{1 + q} \right) + \left( i_b - \frac{q}{1 + q} \right) \left( \frac{L}{1 + q} \right)$$

where $P = \frac{aFN}{cT}$ $E$ is the ratio of the number of flock-born to nucleus-born ewe replacements in the nucleus, and $Q = \frac{E}{bFN}$ is the ratio of nucleus-born to flock ewe replacements. The $i_1, i_2, \ldots$ etc. are the selection differentials corresponding to selected proportions $p, q, \ldots$ and $L(p-k)$ is the genetic difference between nucleus-born and flock-born progeny, born in year $(g-k)$.

Taking $i_1$ and $i_2$, for example, the selection differentials corresponding to the “best” $p$ and “next best” $q$, nucleus-born rams are

$$i_p = \frac{\theta_p}{p} \quad i_q = \frac{(\theta_{p+q} - \theta_p)}{q}$$

where $\theta_p$ is the ordinate of the standard normal curve at the point of truncation of proportion $p$.

Where the unselected drop contained less than 100 individuals, finite population size was allowed for by calculating the selection differentials from finite $N(0,1)$ population tabulated by David $et$ $al$ (1968).

All systems eventually reach a stable situation where the genetic difference between nucleus- and flock-born progeny is constant. All comparisons between systems were in terms of the annual genetic gains at stability.
VI. ACKNOWLEDGMENT

We wish to acknowledge with pleasure the assistance of Miss J. Cherry in the preparation of the figures.

VII. REFERENCES


